Homework 6

1. Find the indefinite integral

   (a)

$$\int \frac{x^4 - 3x^2 + 5}{x^4} dx$$

   (b)

$$\int (\sec x)(\tan x - \sec x)\ dx$$

2. Use the limit process to find the area of the region between the graph of the function and the *y*-axis over the given *y*-interval.

$$f(y) = y^2\ , 0 \leq y \leq 5$$

Sol：

1.

(a)

$$\int \frac{x^4 - 3x^2 + 5}{x^4} dx = \int (1 - 3x^{-2} + 5x^{-4}) dx$$

$$= x - \frac{3x^{-1}}{-1} + \frac{5x^{-3}}{-3} + C$$

$$= x + 3x^{-1} - \frac{5x^{-3}}{3} + C$$

$$= x + \frac{3}{x} - \frac{5}{3x^3} + C$$

(b)

$$\int (\sec x)(\tan x - \sec x)\ dx = \int (\sec x \tan x - \sec^2 x)\ dx = \sec x - \tan x + C$$

2.

$$f(y) = y^2\ , 0 \le y \le 5$$

$$\Delta y = \frac{5 - 0}{n} = \frac{5}{n}$$

$$\mathrm{S(n)} = \sum_{i=1}^{n} f(\frac{5i}{n})(\frac{5}{n})$$

$$= \sum_{i=1}^{n} \left(\frac{5i}{n}\right)^2 \left(\frac{5}{n}\right)$$

$$= \frac{125}{n^3} \sum_{i=1}^{n} i^2$$

$$= (\frac{125}{n^3})(\frac{n(n+1)(2n+1)}{6})$$

$$= \left(\frac{125}{n^2}\right)\left(\frac{(2n^2 + 3n + 1)}{6}\right)$$

$$= \frac{125}{3} + \frac{125}{2n} + \frac{125}{6n^2}$$

$$\mathrm{Area} = \lim_{\mathrm{n}\to\infty} \mathrm{S(n)} = \frac{125}{3}$$